# Squiggle invariants

Here are some property tests for squiggle. I am testing mostly for the mean and the standard deviation. I know that squiggle doesn't yet have functions for the standard deviation, but they could be added.

The keywords to search for are "algebra of random variables".

### Means and standard deviations

Sums

$$mean(f+g) = mean(f) + mean(g)$$

$$std(f+g) = \sqrt{std(f)^2 + std(g)^2}$$

In the case of normal distributions,

$$mean(normal(a,b) + normal(c,d)) = mean(normal(a+c,\sqrt{b^2+d^2}))$$

Subtractions

$$mean(f - g) = mean(f) - mean(g)$$

$$std(f-g) = \sqrt{std(f)^2 + std(g)^2}$$

## Multiplications

$$mean(f \cdot g) = mean(f) \cdot mean(g)$$

 $std(f \cdot g) = \sqrt{(std(f)^2 + mean(f)) \cdot (std(g)^2 + mean(g)) - (mean(f) \cdot mean(g))^2}$ 

#### Divisions

Divisions are tricky, and in general we don't have good expressions to characterize properties of ratios. In particular, the ratio of two normals is a Cauchy distribution, which doesn't have to have a mean.

# To do:

- Provide sources or derivations, useful as this document becomes more complicated
- Provide definitions for the probability density function, exponential, inverse, log, etc.
- Provide at least some tests for division
- See if playing around with characteristic functions turns out anything useful

Probability density functions

TODO

Cumulative density functions

TODO

## Inverse cumulative density functions

TODO